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Dangerous situations in the velocity effect model

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Abstract
This paper investigates the occurrence of dangerous situations (DS) in the velocity effect (VE) model. The VE model is different from the Nagel–Schreckenberg (NS) model and the Fukui–Ishibashi model in that it is based on a non-exclusion process. Two different types of DS—DS caused by stopped cars and DS caused by non-stopped cars—are studied. The results are compared with those from the NS model. It is shown that in the deterministic case, DS caused by stopped cars in the VE model are as likely as those in the NS model provided that one starts from the same random initial conditions. In the non-deterministic case, DS caused by stopped cars in the VE model are qualitatively similar to those in the NS model but quantitatively different. As regards DS caused by non-stopped cars in the VE model, there are none in the deterministic case and there are none when the density is large and positive for small density in the non-deterministic case.

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1. Introduction

Recently, cellular automata (CA) traffic flow models have attracted the interest of a community of physicists [1–4]. The Nagel–Schreckenberg (NS) model is a basic model of traffic flow [5]. It consists of N cars moving in a one-dimensional lattice of L cells with periodic boundary conditions. Each cell may either be empty or be occupied by one car. Each car has an integer velocity v between 0 and the speed limit v_{max}. Let d be the number of empty sites in front of a car; the configuration of N cars is updated by means of four rules applied consecutively, as follows.

(1) Acceleration: $v \to \min(v + 1, v_{max})$. The speed of the vehicle is increased by one, but it remains unaltered if $v = v_{max}$. (2) Slowing down: $v \to \min(d, v)$. If $d < v$, the speed

of the vehicle is reduced to d. This is intended to avoid collisions between the vehicles. (3) Randomization: if $v > 0$, then $v \to v - 1$ with probability p_1. The randomization takes into account the different behavioural patterns of the individual drivers. (4) Motion: the position of a car is shifted by its speed v. These four update rules are applied in parallel to all cars. Iteration over these simple rules already gives realistic results, such as the spontaneous occurrence of traffic jams.

More recently, theoretical and numerical results for dangerous situations (DS) in the framework of the CA models have been reported. In a DS, there will be no accident if every driver is careful enough. Nevertheless, if the drivers are not so careful ($p_2 > 0$; see the following text), an accident may occur. In other words, if one denotes the probability of a DS as P, then a car accident will occur with probability $P \times p_2$.

Boccara *et al* [6] were the first authors to propose conditions for DS in the deterministic[3] NS model. They assume that drivers will probably not respect the safe distance if the speed of the car ahead $v(i+1, t)$ is positive at time t, because drivers expect the speed of the car ahead $v(i+1, t+1)$ to remain positive at time $t+1$. Thus, drivers increase their velocity by one unit with probability p_2; i.e., $v(i, t+1) \to v(i, t+1) + 1$ with probability p_2. It is clear that careless driving will probably result in an accident if the speed of the car ahead $v(i+1, t+1)$ at time $t+1$ becomes zero.

Based on this assumption, Boccara *et al* argue that when the three conditions (i) $0 \leqslant d(i, t) \leqslant v_{\max}$, (ii) $v(i+1, t) > 0$, (iii) $v(i+1, t+1) = 0$ are satisfied, then car i will cause an accident at time $t+1$, with probability p_2. In other words, when the three conditions (i)–(iii) are satisfied, car i is in a DS. If the driver is careful enough ($p_2 = 0$) in this DS, then there is no accident. Nevertheless, if the driver is not so careful ($p_2 > 0$), an accident may occur.

Later, Huang and co-workers [7, 8] as well as Yang and Ma [9] extended the analysis to general situations. They have investigated car accidents for different values of p_1 and $v_{\max}$. A mean field analysis of the occurrence of car accidents as a function of the density and the stochastic braking p_1 in the case of $v_{\max} = 1$ has been carried out [8, 9].

Moreover, Moussa has investigated the effect of the delayed reaction time on the probability of DS in the NS model as well as the DS caused by stopped cars and great deceleration [10]. We also note that the DS problem has been studied in the Fukui–Ishibashi (FI) model [11, 12], in which the main difference is that rule (1) of the NS model changes to $v \to v_{\max}$.

Nevertheless, Jiang *et al* [13] pointed out that car i will not cause an accident at time $t+1$ with probability p_2 when the three conditions (i)–(iii) are satisfied in the deterministic NS model case. They have modified the conditions for the DS to: (1) $v(i, t) \geqslant d(i, t) - 1$ and $0 \leqslant d(i, t) < v_{\max}$; (2) $v(i+1, t) > 0$; (3) $v(i+1, t+1) = 0$. It is shown that when $v_{\max} = 1$, there will be no DS in either deterministic or non-deterministic cases. For $v_{\max} > 1$, the probability of DS covers a two-dimensional region in the deterministic case.

We notice that the above mentioned works were carried out either using the NS model or using the FI model. Both models are based on an exclusion process: at time step $t+1$, car i cannot occupy the site that is occupied by car $i+1$ in time step t; i.e., $x(i, t+1) < x(i+1, t)$.

Recently, Li *et al* [14] presented a new CA model which considers the effect of the velocity of a car on the following car (hereafter, this is referred to as the VE model). In comparison with those of the NS and FI models, the fundamental diagram of the VE model is more consistent with the results measured for real traffic. In addition, the metastable phenomenon

[3] In this paper the terms 'deterministic' and 'non-deterministic' refer only to the evolution rules of the models.

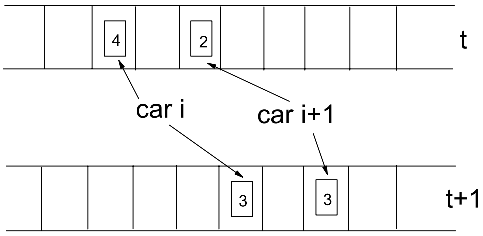

Figure 1. The evolution of the deterministic VE model at time steps t and $t+1$. The speed limit $v_{\max} = 5$. The number denotes the velocity of the car. It can be seen that $x(i, t+1) > x(i+1, t)$; thus the VE model is based on a non-exclusion process.

is reproduced in the deterministic VE model[4]. On the other hand, since the motion of the car ahead is taken into account, the VE model is based on a non-exclusion process. Therefore, it is interesting to analyse what additional effects of DS can be observed in this non-exclusion process.

This paper studies the DS in the VE model. The paper is organized as follows. In section 2, the VE model is briefly reviewed and the conditions for DS in the VE model are introduced. The numerical simulations are carried out and compared with ones based on the NS model in section 3. The conclusions are given in section 4.

2. Conditions for DS in the VE model

Before the conditions for DS in the VE model are introduced, we discuss the conditions for DS in the NS model. In [13], the probability that the three modified conditions (1)–(3) are met is denoted as P_{ac}^1; the probability that the car is in a DS is denoted as P_{ac}^2. It is pointed out that in the deterministic NS model, $P_{ac}^1 = P_{ac}^2$, while in the non-deterministic NS model, $P_{ac}^2 = P_{ac}^1 \times (1 - p_1)$.

We note that in the deterministic NS model, the three modified conditions (1)–(3) are equivalent to the three conditions (a) $v(i, t+1) < v_{\max}$ and $d(i, t+1) = 0$; (b) $v(i+1, t) > 0$; (c) $v(i+1, t+1) = 0$. We also note that in the NS model, the condition $d(i, t+1) = 0$ implies $v(i+1, t+1) = 0$ because the NS model is based on an exclusion process. Therefore, the conditions (a)–(c) are equivalent to the conditions (a) and (b). Thus, if we denote the probability that conditions (a) and (b) are met as P_{ac}^3, then $P_{ac}^3 = P_{ac}^1$ in the deterministic NS model. In contrast, in the non-deterministic NS model, $P_{ac}^3 = P_{ac}^1 \times (1 - p_1)$. Thus, $P_{ac}^2 = P_{ac}^3$ in both the deterministic NS model and the non-deterministic NS model. In view of this, one does not need to distinguish the deterministic NS model from the non-deterministic NS model in the calculations on DS. In the following, similar conditions for DS are adopted in the VE model.

Next we briefly review the VE model. In the VE model, the slowing down rule is different from that of the NS model: it reads $v \to \min(d(i, t) + v'(i+1, t), v)$ where $v'(i+1, t)$ is the virtual velocity of the car ahead and it is determined by $v'(i+1, t) = \min(v_{\max} - 1, v(i+1, t), \max(0, d(i+1, t) - 1))$. The virtual velocity represents the effect of anticipation by the driver, which is obtained by applying the velocity update rule of the NS model to the $i+1$th car and considering the random delay. Due to the introduction of virtual velocity, the VE model is a based on a non-exclusion process; see, e.g., figure 1.

[4] We should note here that these NS based CA models cannot describe real congestion patterns at motorway/freeway bottlenecks. On the basis of three-phase traffic theory, Kerner *et al* presented a new CA model which can reproduce the real congestion patterns; see, e.g., [15] and references therein.

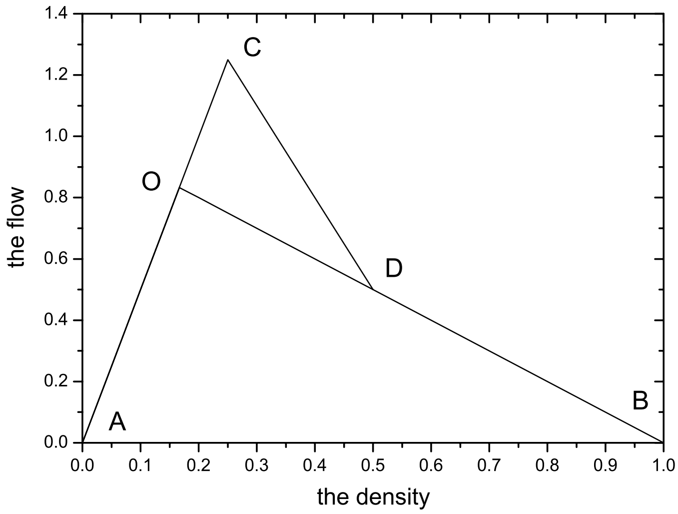

Figure 2. The fundamental diagrams of the deterministic VE model. The speed limit $v_{\max} = 5$.

Due to the VE model being based on a non-exclusion process, $d(i, t + 1) = 0$ does not imply $v(i + 1, t + 1) = 0$. Therefore, two different types of DS need to be discussed:

- The first type of DS is caused by stopped cars, the conditions for which are (a1) $v(i, t + 1) < v_{\max}$ and $d(i, t + 1) = 0$; (b1) $v(i + 1, t) > 0$; (c1) $v(i + 1, t + 1) = 0$. We assume that the probability that conditions (a1)–(c1) are met is P_{ac}^4. The probability of the first type of DS is also P_{ac}^4.
- The second type of DS is caused by non-stopped cars, the conditions for which are (a2) $v(i, t + 1) < v_{\max}$ and $d(i, t + 1) = 0$; (b2) $v(i + 1, t) > 0$; (c2) $v(i + 1, t + 1) > 0$. We assume that the probability that conditions (a2)–(c2) are met is P_{ac}^5. The probability of the second type of DS is also P_{ac}^5.

3. Numerical simulation

In the simulations, a car accident defined as a car that hits the car ahead does not really happen. We are looking for DS on the road and take them as indicators for the occurrence of car accidents. In the simulations, it is the VE and/or the NS model that is used; the velocity increase due to careless driving is not actually carried through.

3.1. DS caused by stopped cars

In this subsection we study P_{ac}^4. First, we neglect stochastic driving behaviour and study P_{ac}^4 in the deterministic case. For $v_{\max} = 1$, P_{ac}^4 is always zero. This is easy to understand because the virtual velocity in the VE model is zero for $v_{\max} = 1$ and the VE model reduces to the NS model.

Next we study the cases with $v_{\max} > 1$. Without loss of generality, we choose $v_{\max} = 5$. Before the results for P_{ac}^4 are shown, we have a look at the fundamental diagrams of the VE model in figure 2. One can see the metastable phenomenon in the VE model. The branches OC and CD start from initially homogeneous (or nearly homogeneous) distributions of cars and the branch BO starts from a random distribution of cars.

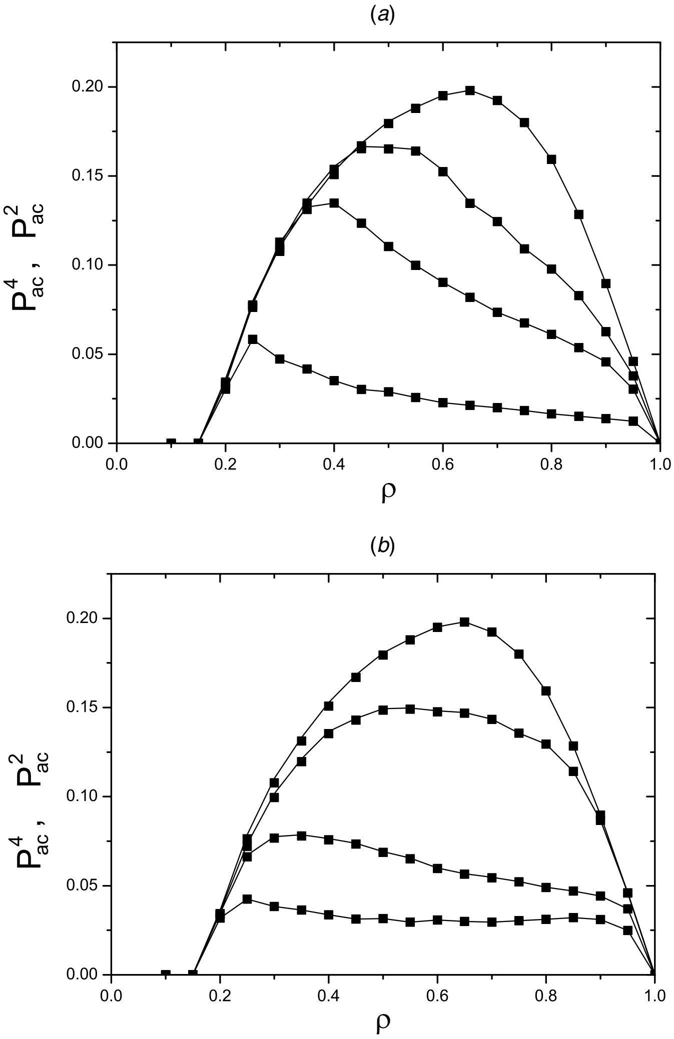

Figure 3. The dependence of P_{ac}^4 (solid line) and P_{ac}^2 (scattered points) on the density and the initial condition in the deterministic case. The speed limit $v_{\max} = 5$. In (*a*), from top to bottom, $p_3 = 0.1, 0.02, 0.01, 0.002$; in (*b*), from top to bottom, $p_3 = 0.1, 0.7, 0.9, 0.95$.

For the traffic situations on branches AC and CD, $P_{ac}^4 = 0$. This is due to there being no stopped car. We focus on traffic situations on the branch BO. The simulations are carried out with different initial configurations, which are prepared as in reference [13]. We assume that at $t = -t_0$, the traffic is in a 'megajam'. Then the cars move, obeying the non-deterministic NS model with the stochastic randomization p_3.[5] The system evolves from $t = -t_0$ to $t = 0$ and the traffic condition of the system at $t = 0$ is used as the initial configuration. From $t = 0$, the system will evolve according to the deterministic VE model.

We show the results in figure 3. In the simulations, $L = 1000$, $t_0 = 20\,000$ and the data from $t = 0$ to $t = 10\,000$ are discarded to let the transient die out. An average over 100 different random seeds is taken for each data point. From figure 3, one can see that P_{ac}^4 covers

[5] Here we emphasize that p_3 is used only for the generation of initial conditions.

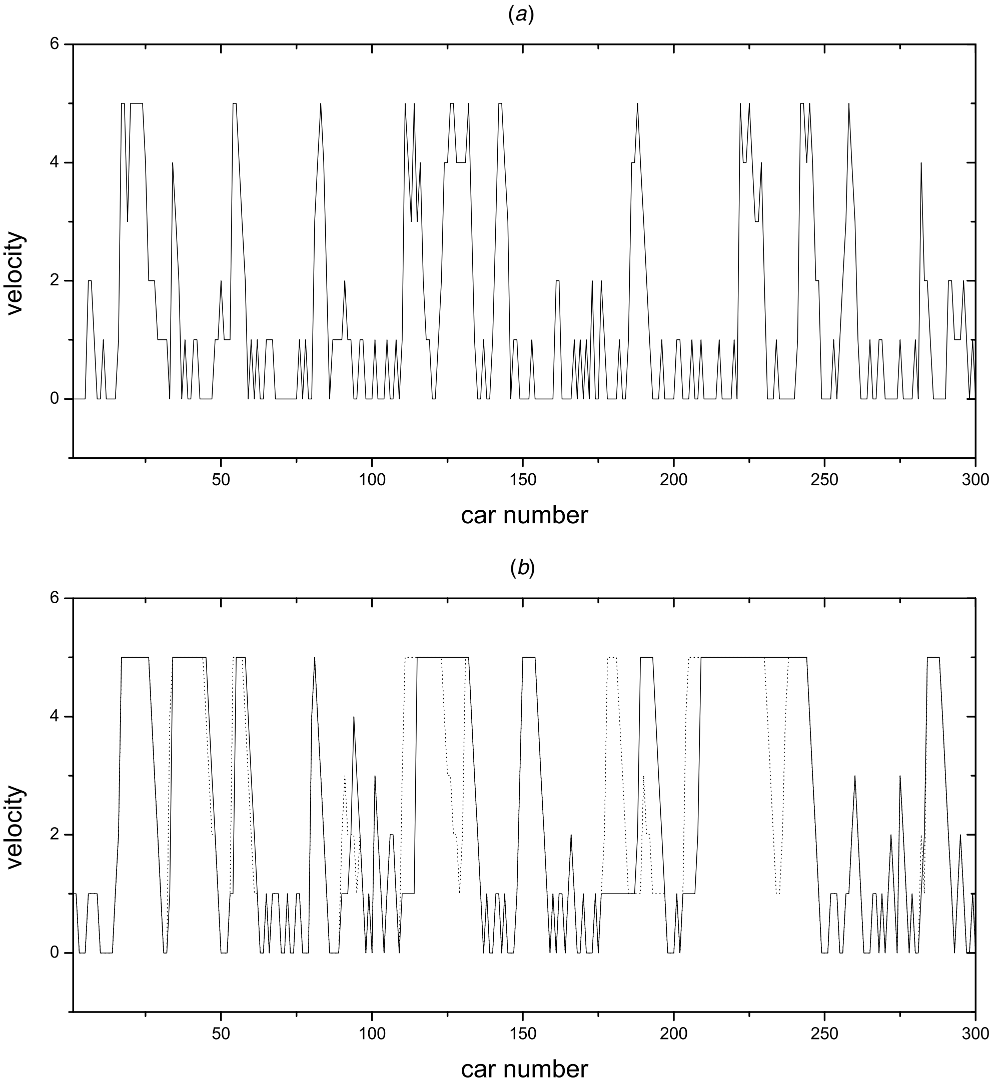

Figure 4. (*a*) The initial velocity profile; (*b*) the stationary states evolved from (*a*) in the deterministic VE case (solid line) and the NS case (dotted line).

a two-dimensional region. It depends not only on the density of the system but also on the value of p_3. It is more interesting that P_{ac}^4 equals P_{ac}^2, the probability of DS in the NS model, provided that the same initial conditions (i.e., the same values of p_3) are used.

In figure 4, we show the stationary velocity profiles of the deterministic NS model and the VE model that start from the same initial conditions. It can be seen that their velocity profiles are nearly the same except for a few cars that have positive velocities. The nearly identical profiles lead to the same values of P_{ac}^4 and P_{ac}^2. The virtual velocity does not contribute to the probability of DS in the deterministic case. This is explained as follows. If $v(i+1, t+1) = 0$, one has $d(i+1, t) = 0$. Thus, $v'(i+1, t) = 0$. Consequently, car i will move as in the NS model.

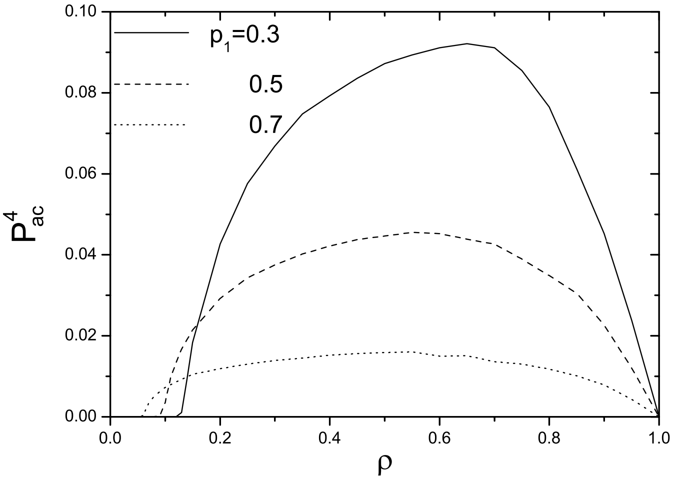

Figure 5. The dependence of P_{ac}^4 on the density in the non-deterministic case. The speed limit $v_{\max} = 5$.

Next we proceed to study P_{ac}^4 in the non-deterministic VE model. Similarly, $P_{ac}^4 = 0$ for $v_{\max} = 1$ because the VE model reduces to the NS model. For $v_{\max} > 1$, we still choose the typical value $v_{\max} = 5$. As in the non-deterministic NS model, P_{ac}^4 is independent of the initial configuration (see figure 5). When the density is below a critical density ρ_c, there is no DS. For the density $\rho > \rho_c$, P_{ac}^4 first increases with ρ. After it reaches the maximum, it decreases with ρ. Moreover, both the maximum value of P_{ac}^4 and the critical density ρ_c decrease with increase of p_1. These properties are qualitatively similar to those in the NS model.

In figure 6, a quantitative comparison of P_{ac}^4 and P_{ac}^2 is shown. One can see that for small p_1, $P_{ac}^4 < P_{ac}^2$ for intermediate density and $P_{ac}^4 = P_{ac}^2$ for large density. Moreover, the critical density in the VE model is larger than that in the NS model. In figure 7, we show the spacetime plots of the VE model and the NS model. One can see that for $\rho = 0.3$, the jams in the VE model are less fragmented than those in the NS model (see also [16]). This leads to a smaller value of the probability of DS in the VE model for intermediate density. In contrast, for large density, there is almost no difference between the spacetime plots of the VE model and the NS model (not shown). For this case, the velocities of the cars are small and the virtual velocities can be approximately neglected. This leads to equal values of the probability of DS in the VE model and that in the NS model for large density.

With increase of p_1, the qualitative difference between the spacetime plots of the VE model and the NS model in the intermediate density range gradually disappears. As a result, the difference between P_{ac}^4 and P_{ac}^2 gradually decreases with increase of p_1 in the intermediate density range (cf figure 6(*b*)). For $p_1 = 0.7$, the difference between P_{ac}^4 and P_{ac}^2 is approximately zero (figure 6(*c*)).

3.2. DS caused by non-stopped cars

In this subsection we study P_{ac}^5. First, we neglect stochastic driving behaviour and study P_{ac}^5 in the deterministic case. Our simulations show that $P_{ac}^5 = 0$ in the deterministic case whatever $v_{\max}$ is. This is explained as follows. Since

$$v'(i+1, t) = \min(v_{\max} - 1, v(i+1, t), \max(0, d(i+1, t) - 1))$$

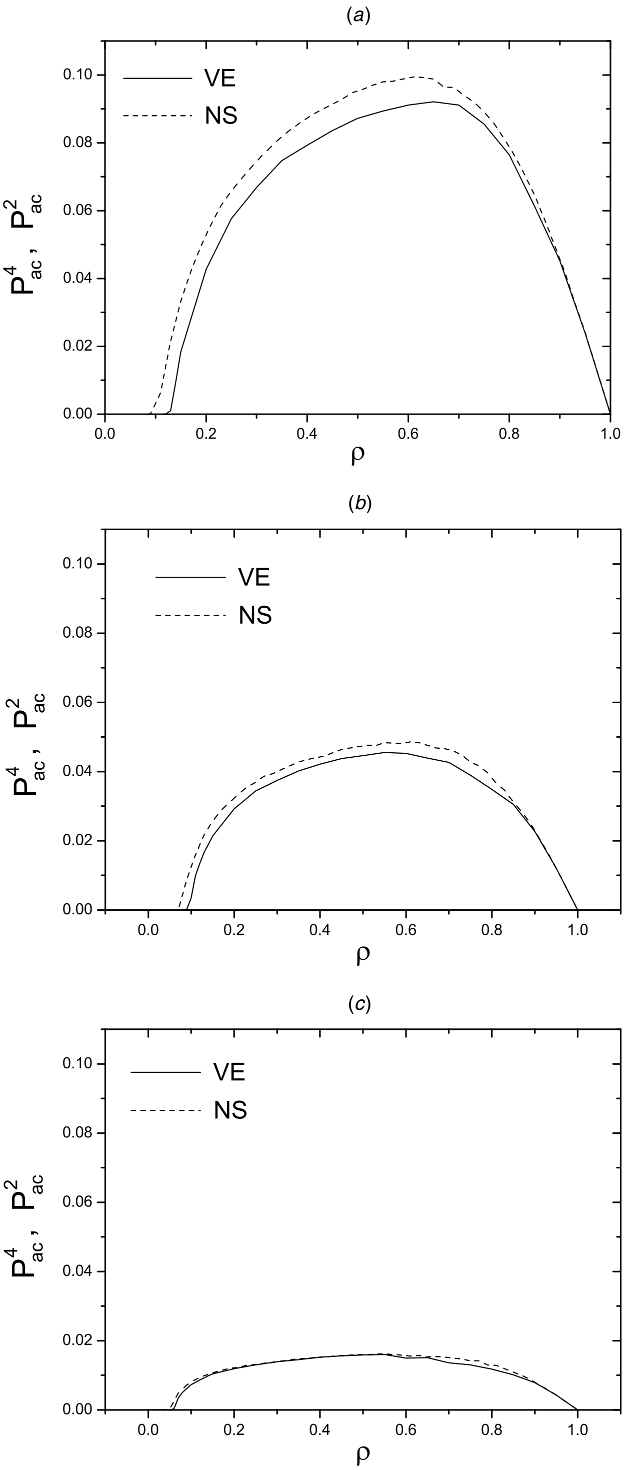

Figure 6. The comparison of P_{ac}^4 with P_{ac}^2. (*a*) $p_1 = 0.3$; (*b*) $p_1 = 0.5$; (*c*) $p_1 = 0.7$. The speed limit $v_{\max} = 5$.

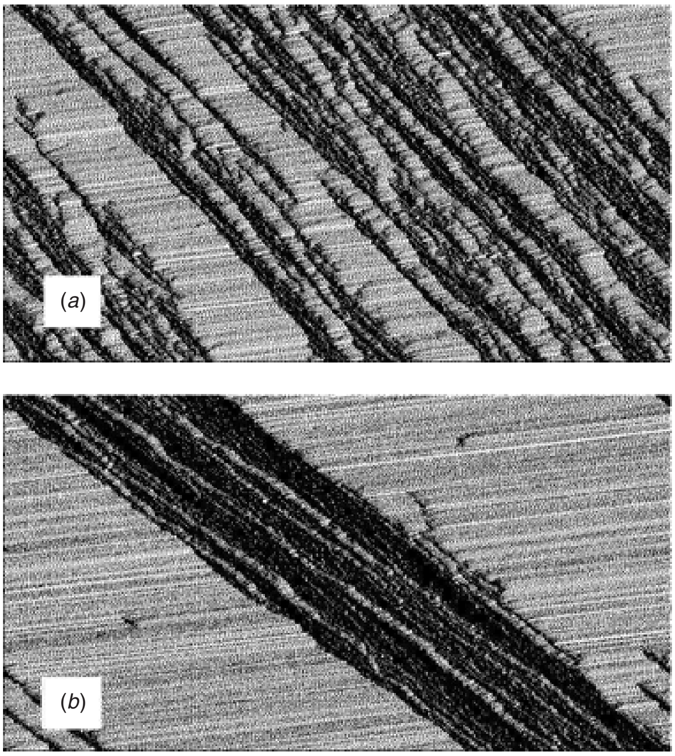

Figure 7. The spacetime plots of (*a*) the NS model and (*b*) the VE model. $p_1 = 0.3, \rho = 0.3, v_{\max} = 5$. The cars are moving from the left to the right, and the vertical direction (up) is (increasing) time. Each horizontal row of dots represents the instantaneous positions of the vehicles moving towards the right while the successive rows of dots represent the positions of the same vehicles at the successive time steps.

and

$$v(i+1, t+1) = \min(v_{\max}, v(i+1, t) + 1, d(i+1, t) + v'(i+2, t)),$$

we have $v'(i+1, t) < v(i+1, t+1)$ if $d(i+1, t) \geqslant 1$. As a result,

$$v(i, t+1) = \min(v_{\max}, v(i, t) + 1, d(i, t) + v'(i+1, t)) < d(i, t) + v(i+1, t+1).$$

This means that for $d(i, t+1) > 0$, conditions (a2) cannot be met. On the other hand, if $d(i+1, t) = 0$, we have $v'(i+1, t) = 0$. Condition (c2) requires $v(i+1, t+1) > 0$; thus,

$$\begin{aligned} v(i, t+1) &= \min(v_{\max}, v(i, t) + 1, d(i, t) + v'(i+1, t)) \\ &= \min(v_{\max}, v(i, t) + 1, d(i, t)) < d(i, t) + v(i+1, t+1). \end{aligned}$$

This also means that for $d(i, t+1) > 0$, conditions (a2) cannot be met.

Next we proceed to study P_{ac}^5 in the non-deterministic VE model. It is obvious that $P_{ac}^5 = 0$ for $v_{\max} = 1$ because the VE model reduces to the NS model and, in the NS model, conditions $v(i+1, t+1) > 0$ and $d(i, t+1) = 0$ cannot be met simultaneously.

For $v_{\max} > 1$, the situation is different. In figure 8, we show the results for $v_{\max} = 2$. One can see that P_{ac}^5 is relatively small: it is of the order of 10^{-3}. Moreover, unlike the case for P_{ac}^4, $P_{ac}^5 = 0$ when the density is large. This is because for large density, no car can move in two successive time steps. P_{ac}^5 is also different from P_{ac}^4 in that it is positive for small

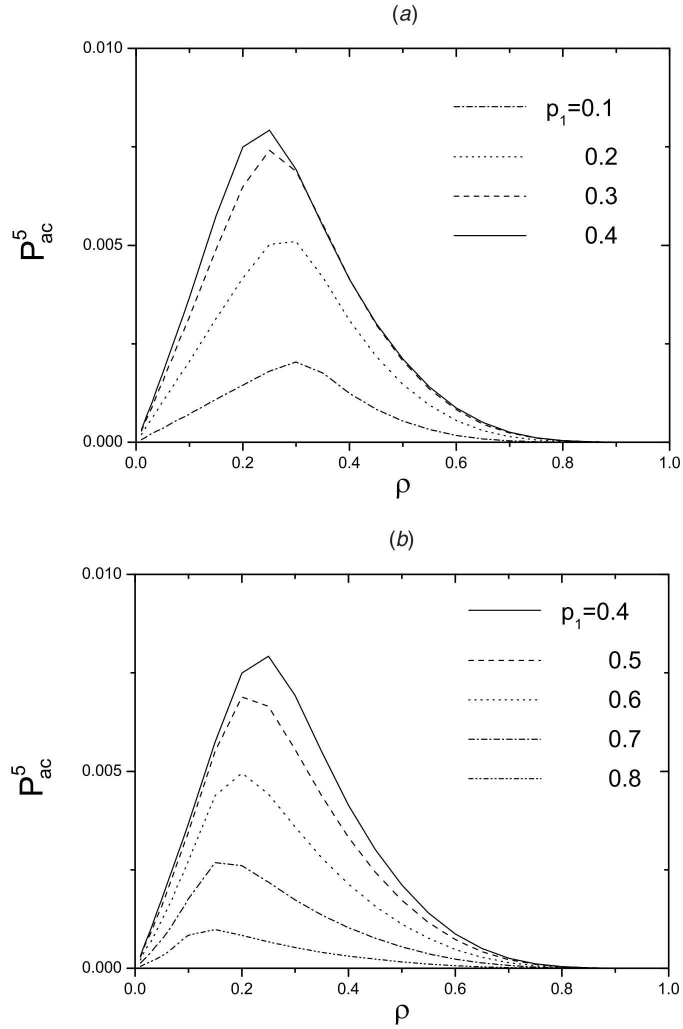

Figure 8. The dependence of P^5_{ac} on the density in the non-deterministic case. The speed limit $v_{max} = 2$.

density. P^5_{ac} firstly increases with the density; after the maximum is reached, it decreases with the density. When p_1 is small, P^5_{ac} increases with increase of p_1 (figure 8(*a*)); but when p_1 is large, it decreases with increase of p_1 (figure 8(*b*)). For larger v_{max}, similar properties of P^5_{ac} can be obtained.

4. Summary

In this paper, we have investigated the occurrence of DS in the VE model. The VE model is different from the NS model and FI model in that it is based on a non-exclusion process. Two different types of DS—DS caused by stopped cars $\left(P^4_{ac}\right)$ and DS caused by non-stopped cars $\left(P^5_{ac}\right)$—are studied.

It is shown that in the case of $v_{max} = 1$, $P^4_{ac} = 0$ and $P^5_{ac} = 0$ because the VE model reduces to the NS model. For $v_{max} > 1$ and in the deterministic case, P^4_{ac} covers a two-dimensional region and is equal to P_{ac}^2 provided that one starts from the same initial conditions. In the non-deterministic case, P_{ac}^4 is qualitatively similar to P_{ac}^2 but quantitatively different. As for P_{ac}^5, it is zero in the deterministic case whatever $v_{\max}$ is. In the non-deterministic case, it is relatively small (of the order of 10^{-3}) for $v_{\max} > 1$. It is different from P_{ac}^4 in that $P_{ac}^5 = 0$ when the density is large and it is positive for small density.

Acknowledgment

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References

[1] Schreckenberg M and Wolf D E (ed) 1998 *Traffic and Granular Flow '97* (Singapore: Springer)
Helbing D, Herrmann H J, Schreckenberg M and Wolf D E (ed) 2000 *Traffic and Granular Flow '99* (Berlin: Springer)
[2] Chowdhury D, Santen L and Schadschneider A 2000 *Phys. Rep.* **329** 199
[3] Helbing D 2001 *Rev. Mod. Phys.* **73** 1067
[4] Nagel K *et al* 2003 *Oper. Res.* **51** 681
[5] Nagel K and Schreckenberg M 1992 *J. Phys.* I **2** 2221
[6] Boccara N, Fuks H and Zeng Q 1997 *J. Phys. A: Math. Gen.* **30** 3329
[7] Huang D W and Wu Y P 2001 *Phys. Rev.* E **63** 022301
[8] Huang D W and Tseng W C 2001 *Phys. Rev.* E **64** 057106
[9] Yang X Q and Ma Y Q 2002 *Mod. Phys. Lett.* B **16** 333
[10] Moussa N 2003 *Phys. Rev.* E **68** 036127
[11] Huang D W 1998 *J. Phys. A: Math. Gen.* **31** 6167
[12] Yang X Q and Ma Y Q 2002 *J. Phys. A: Math. Gen.* **35** 10539
[13] Jiang R, Wang X L and Wu Q S 2003 *J. Phys. A: Math. Gen.* **36** 4763
[14] Li X B, Wu Q S and Jiang R 2001 *Phys. Rev.* E **64** 066128
[15] Kerner B S 2002 *Phys. Rev.* E **65** 046138
Kerner B S *et al* 2002 *J. Phys. A: Math. Gen.* **35** 9971
[16] Jia B, Jiang R and Wu Q S *Int. Mod. Phys.* C at press